

1997

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## Citation of this paper:

Bhatia, Kul B.. "Specific Inputs, Value-Added, and Production Linkages in Tax-Incidence Theory." Department of Economics Research Reports, 9714. London, ON: Department of Economics, University of Western Ontario (1997).

4 88 06

ISSN:0318-725X  
ISBN:0-7714-2034-X

**RESEARCH REPORT 9714**

**Specific Inputs, Value-Added, and Production  
Linkages in Tax-Incidence Theory**

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November 1997

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# **SPECIFIC INPUTS, VALUE-ADDED, AND PRODUCTION LINKAGES IN TAX-INCIDENCE THEORY**

**Kul B. Bhatia\***

## **Abstract**

A general equilibrium framework is developed for analyzing the role of immobile factors of production which produce inputs for other sectors. The production process and the cross-sector connections are explicitly specified, and tax-incidence propositions are compared with those in related models. Numerical examples, based on a consistent data set for the U.S. economy, illustrate the results and highlight the difficulties that arise in defining equivalent specifications, analytically and empirically. Goods mobility offsets some effects of factor immobility, but the computed tax elasticities are rarely the same as in mobile-factors-only models. The Marshallian short-run-long-run distinction, blurred somewhat by production linkages, does not disappear.

**Keywords:** sector-specific inputs; tax incidence

***JEL* classification:** H22

## 1. Introduction

In an economy with two sectors, each producing a final good with the help of a mobile factor (labor) and a sector-specific input (capital), a Harberger-type neoclassical model leads to the well-known tax-incidence proposition that a partial output tax, or a tax on labor in one industry, will hurt labor throughout the economy and benefit capital in the untaxed sector. This result, formally derived by McClure (1971) in a 3x2 final-goods-only (fgo) model is typical of a class of models which deal with topics such as factor immobility, regional incidence of taxes, and short-run and long-run considerations (Bhatia (1987)).

These models generally include primary factors of production (undeveloped land, unskilled labor), but even when a value-added component is indicated - some enhancement over their primary state (on-the-job instruction for workers) or old-fashioned production (plant and equipment) - the value-adding process is rarely modeled, and the immobile inputs are not assigned any role outside their specific domains. While this structure is appropriate in some cases (licensing requirements, mobility restrictions, location preferences), many situations arise in which the immobile factors of production in one sector are involved in producing goods and services that become specialized inputs in other sectors. There are examples galore: synthetic fertilizers produced in the manufacturing sector and used in agriculture; accounting, advertising, and insurance services provided by the tertiary sector for the rest of the economy; latex and other raw materials gathered, mined, or grown in the primary sector for further processing or production elsewhere in the economy; in fact, many items commonly classified in the primary category - crude oil, iron ore, even land in some instances - would need a contribution from other inputs, from other sectors sometimes, before becoming productive themselves. A wide variety of inputs thus can be regarded as "produced" because, at the very least, they will require some handling, packaging, or

transportation before being put to work. Once possibilities of CAD/CAM and custom-made software and hardware are recognized, the number of such inputs and their cross-sector connections potentially can be very large indeed. The extensive tax literature which deals with immobile factors of production in one context or another, however, hardly makes a mention of produced specific inputs (psi's) and their production linkages.

This paper develops a framework in which the value-adding process and the contribution of psi's to all sectors of the economy are explicitly modeled. The analysis begins with a minimal modification of the 3x2 fgo setup and replaces one of its immobile inputs by a specific input produced in the other sector. But that is enough to dislodge the tax-incidence results noted above: a selective production tax, rather than hurting the mobile factor, may actually benefit it, and the specific factor in the untaxed sector may lose. Moreover, a partial tax on the mobile factor, which mimics an output tax in the 3x2 fgo model (subject to some exceptions), may have very different effects in this specification. In spite of the sector-specific inputs, the tax-incidence propositions are often similar to those in a typical mobile-factors-only (mfo) model.

Apart from questions of tax incidence, two related issues which arise in many areas of economic analysis and policy will also be considered: First, the more interesting propositions emerge when one sector produces an input for another, as in the fertilizer/latex examples. Can such cross-sector connections offset or eliminate the effects of factor immobility if some inputs are physically immobile, or prohibitively costly to move? Second, the Marshallian short-run-long-run distinction based on factor mobility has a long and venerable history. All inputs are mobile in the long run, although some of them may not be so in the short run. If immobile factors of production can produce mobile inputs, is the short run still a distinct and meaningful concept? A logical approach to such questions is to set up comparable models, but which models may be regarded as

equivalent is often a complex, methodological issue involving theoretical and empirical considerations. One contribution of this paper is to show that alternative specifications which may be viewed as analytically equivalent will not always generate the same outcomes, and additional complications may arise when actual data are involved.

The analytical framework is laid out in the next section, and questions of tax incidence are taken up in Section 3. The results are compared with those in mobile-factor models in Section 4 where the issue of model equivalence and related questions are also taken up. Some numerical illustrations based on computable general equilibrium (cge) techniques are presented in Section 5, and the conclusions are summarized in Section 6.

## **2. The Analytical Framework**

To formalize an example introduced in the previous section, it is assumed that the economy produces two final goods - food ( $X_1$ ) and rubber products ( $X_2$ ). Food is produced in the agricultural sector which also supplies latex ( $X_3$ ) to be used specifically in manufacturing  $X_2$ . Land ( $K$ ) is the specific input in agriculture, and labor ( $L$ ), the only mobile factor, is employed in all three activities. Alternatively, one sector may be the software sector, employing specialized talent ( $K$ ) to produce computer games for home entertainment ( $X_2$ ), and computer assisted designs (CAD) for industrial use ( $X_3$ ). Among other assumptions, the two primary factors,  $L$  and  $K$ , have exogenous endowments, markets are competitive with no excess demand or supply anywhere, and the three production functions are linear homogeneous. Denoting input-output coefficients by  $a_{ij}$ , the full employment conditions can be written as:

$$R_{L1}X_1 + a_{L2} X_2 + a_{L3} X_3 = \bar{L} \quad (1)$$

$$a_{K2} X_2 + a_{K3} X_3 = \bar{K} \quad (2)$$

$$a_{31} X_1 = X_3 \quad (3)$$

Combining equation (3) with the other two, we get:

$$R_{L1}X_1 + a_{L2} X_2 = \bar{L} \quad (4)$$

$$a_{K3}a_{31} X_1 + a_{K2} X_2 = \bar{K} \quad (5)$$

Here  $a_{L1}$  represents total usage of labor by  $X_1$  - direct as well as through  $X_3$ , and because of this linkage, farm land indirectly becomes an input for  $X_1$  also, although it is directly used only where it is physically located, in agriculture.

This framework can be regarded as a two sector model with two mobile factors - labor, physically mobile, and farm land effectively so, through  $X_3$ . The two final goods thus can be ranked in terms of their land-labor ratios, which is a key feature of mobile-factor models and will figure prominently in many of the results to be derived below. Farm land, however, is physically immobile between the two sectors, and that points in the direction of McLure's specification. The two models have the same dimension, 3x2, although the numbers of factors and goods are reversed (two primary factors and three goods here). The psi specification nonetheless follows if one of McLure's immobile factors of production is replaced by a psi produced in the other sector, but relative factor intensities are generally not comparable across industries in McLure-type models because the industry-specific inputs are unique. On the whole, considering these similarities and differences, one should expect a mixture of results--some similar to those in mfo models, others closer to the tax-incidence propositions in the 3x2 fgo specifications, and a few hybrids too.

Location of inputs, their ultimate usage, and the value-adding process for psi's play rather important roles in this framework. In a situation where  $X_2$  uses  $X_3$ , for instance, labor will be the

only input common to the two final goods, so their factor intensities could not be compared. This complication will not arise in our examples if latex production is shifted to the manufacturing sector, or some software specialists move there; in that case, there will be two mobile factors and an intermediate good, and the present analysis, with some modification, will apply. If the latex/food example appears simplistic, slight alteration of the equations can portray other possibilities: e.g. only one agricultural commodity - corn for eating as well as further processing in the manufacturing sector.  $X_3$  then can be regarded as the portion of the agricultural output used in the other sector, a one-way input-output model. Different ways of producing specific inputs can also be considered -  $X_3$  requiring some  $X_1$  in its production, or the manufacturing sector also producing a second output (fertilizer) for use in  $X_2$ , and so on.

Returning to the original specification, and letting asterisks indicate proportional changes, total differentiation of equations (4) and (5) leads to:

$$\lambda_{L1} X_1^* + \lambda_{L2} X_2^* = -\lambda_{L1} R_{L1}^* - \lambda_{L2} a_{L2}^* \quad (6)$$

$$\lambda_{K1} X_1^* + \lambda_{K2} X_2^* = -\lambda_{K2} a_{K2}^* - \lambda_{K3} (a_{K3}^* + a_{31}^*) \quad (7)$$

where  $\lambda_{ij}$  represents the proportion of the  $i^{\text{th}}$  input used directly or indirectly by the  $j^{\text{th}}$  activity. Thus  $(\lambda_{L1} + \lambda_{L2}) = (\lambda_{K1} + \lambda_{K2}) = 1$ . For minimizing unit costs, input choices depend on relative input prices, so  $R_{L1}^*$  and the  $a_{ij}^*$ 's can be solved in terms of input prices and elasticities of substitution ( $\sigma$ ). For instance,  $a_{L2}^* = -\rho_{K2}(w^*-r^*)\sigma^2$ , and  $R_{L1}^* = -(w^*-r^*)\rho_{31}\rho_{K3}(\rho_{L1}\rho_{K3}\sigma^1 + \rho_{L3}\sigma^3)$ , which reflects substitution possibilities in psi production as well as in the using sector ( $X_1$ ). The  $\rho$ 's denote direct input shares and the  $\theta$ 's include indirect usage as well. For example,  $\rho_{K2} = (wK_2/p_2X_2)$ , and  $\theta_{L1} = \rho_{L1} + \rho_{31}\rho_{L3}$ .<sup>1</sup>

The zero-profit conditions in the final goods markets can be written as:

$$\theta_{L1}w^* + \rho_{K1}r^* = p_1^* \quad (8)$$



and 
$$\rho_{L2}w^* + \rho_{K2}r^* = p_2^* \quad (9)$$

The presence of  $\theta_{L1}$  indicates that equation (8) already incorporates the zero-profit condition for  $X_3$ . A demand function,  $X_1^* = \varepsilon(p_2^* - p_1^*)$  completes the analytical specification of the model.<sup>2</sup>

Various taxes will modify one or more of the above equations: in  $X_2$ , a selective production tax will add  $T_2^*$  to the left-hand side of (9), and a partial tax on labor ( $t_{L2}$ ) will change the solutions for  $a_{K2}^*$  and  $a_{L2}^*$  as well, and these changes will be carried through to the demand function too. Taxes in  $X_1$  will have similar effects on the equations concerned. The incidence of a given tax in this type of models depends on what happens to the relative income shares of the primary factors of production, i.e., on changes in input prices because of the full-employment assumption. The model, therefore, is solved for  $(w^* - r^*)$ , for which a two-step process seems convenient: determine  $X_1^*$  from the supply-side equations after incorporating the relevant tax terms, and then equate it to the corresponding change in demand. Choosing  $w$  as the numeraire (so that  $w^* = 0$ ) and setting initial prices to unity will simplify some of the derivations.

### 3. Tax Incidence

We shall consider a selective output tax and a partial tax on labor because these taxes provide an interesting contrast with the results in the tax literature, and many other taxes can be analyzed in an analogous manner.

#### 3.1 Selective output taxes

When a tax on the output of  $X_1$  ( $T_1$ ) is levied, equation (8) will change to:

$\theta_{L1}w^* + \theta_{K1}r^* + T_1^* = p_1^*$ , and a new term,  $-\varepsilon T_1^*$ , will appear in the demand function. Equations (6) and (7) will remain unchanged and can be solved for the proportional change in the supply of  $X_1$ :

$$X_1^* = - (\theta_{K3} S_1 \sigma^1 + S_2 \sigma^2 + S_3 \sigma^3) r^* / |\lambda| \quad (10)$$

where the  $S$ 's are positive, being sums of  $\lambda$ 's,  $\rho$ 's,  $\theta$ 's (all positive fractions), and  $\sigma$ 's are defined to be non-negative. For instance,  $S_1 = (\lambda_{L2} \lambda_{K3} + \lambda_{K2} \lambda_{L1} \rho_{31} \theta_{K3} / \theta_{L1}) \rho_{L1} \theta_{K3}$ . The determinant,  $|\lambda|$ , reflects the factor intensities of the two final goods,  $X_1$  and  $X_2$ , based on the total usage of each input. Thus, if  $X_1$  is relatively labor intensive, after taking into account the land and labor employed in producing  $X_3$ ,  $|\lambda| > 0$ .

Equation (10) and the demand function (with the tax-term added) yield:

$$r^* = \varepsilon |\lambda| T_1^* / D \quad (11)$$

where  $D = \varepsilon (\theta_{K2} - \theta_{13} \theta_{K3}) |\lambda| + \theta_{K3} S_1 \sigma^1 + S_2 \sigma^2 + S_3 \sigma^3$ . The  $S$ 's and the  $\sigma$ 's have been discussed above; the first term in  $D$  is new, and it will be positive because  $\varepsilon > 0$ , and  $|\lambda|$  and the term within the parentheses will have the same sign (both negative, for instance, when  $X_1$  is relatively land-intensive).  $D$ , therefore, will be generally positive, and the sign of  $r^*$  will depend on the numerator of equation (11).

It should be noted at the outset that when the demand for  $X_1$  is completely inelastic, or if any of the elasticities of substitution is sufficiently large,  $r^* = 0$ . With inelastic demand, none of the adjustments likely to follow the imposition of a selective output tax will take place, and for large  $\sigma$ 's, any excess supply or demand for  $K$  or  $L$  caused by a change in outputs can be accommodated without any variation in the wage-rental ratio. These outcomes are well known in McLure-type fgo models, indeed in the broader tax literature as well, so further elaboration is not needed. We turn, instead, to new results.

*Result 1: The incidence of a selective output tax depends on relative factor intensities; the primary specific input benefits ( $r^* > 0$ ) only if the taxed industry is relatively labor intensive.*

The tax leads to a reduction in the output of  $X_1$  thereby reducing the demand for labor as well as for its specific input,  $X_3$ . If the taxed industry is relatively labor intensive, there will be an excess supply of labor in the economy, and the wage-rental ratio will fall to restore full employment.

This straightforward result is remarkable in that, by itself, it bespeaks no sector-specific inputs, produced or primary; it rather reads like a proposition from tax models with all mobile factors of production. The similarity is not fortuitous because, as noted earlier in connection with the full-employment conditions, both inputs, in effect, are mobile even though farm land cannot be physically moved to the manufacturing sector. Land-labor ratios of the two final goods can be compared, and they are sufficient to determine the direction of change in the wage-rental ratio as in the conventional 2x2 framework with all mobile inputs.

In 3x2 fgo models, the primary specific input in  $X_2$  always benefits from a tax of this sort simply because its land-labor ratio declines due to an influx of labor from  $X_1$ . Here  $X_3$  complicates the adjustment process because, in addition to the movement of labor, land will be reallocated between  $X_2$  and  $X_3$ . The key element in the analysis is the relative land-labor ratio of the taxed industry. If  $X_1$  is relatively land-intensive, although some labor is released as this industry contracts, there will be a bigger decline in the demand for land, and  $r^*$  will be negative.

An analogous reasoning applies to a partial output tax on  $X_2$  (in this case,  $r^* = -\epsilon|\lambda|/D$ , though). Since  $X_2$  is the taxed industry now, when it is relatively labor intensive,  $|\lambda| < 0$ . Because of the tax, food production declines, but the output of  $X_1$  increases, bringing in its wake a greater demand for  $X_3$ , eventually leading to a higher rental for land. There is nothing comparable to this outcome in the 3x2 fgo framework where a specific factor cannot possibly benefit from a tax on the output of a good it helps directly produce.

### 3.2 Partial Factor Taxes

A factor tax induces cost-minimizing firms to substitute away from the taxed input, and their ability to do so depends on the relevant elasticity of substitution. For a tax on labor directly employed in  $X_1$ , therefore,  $\sigma^1$  will be of special interest. For an output tax on  $X_1$ , this elasticity affected the magnitude of  $r^*$ , as a component of  $D$ . Here it can also change the sign of  $r^*$  and lead to a situation in which, unlike Result 1,  $r^* > 0$  even when the taxed industry is relatively land-intensive.

Note, first, that this tax will add a new term,  $\rho_{L1} t_{L1}^*$ , to the left-hand side of equation (8), where  $t_{L1}^*$  is the tax (in percent terms) on labor in  $X_1$  and  $\rho_{L1}$  is the share of labor directly employed there. The demand function too will be modified accordingly, and all the expressions dealing with input choices in  $X_1$  -  $a_{31}^*$ ,  $a_{L1}^*$ , and  $R_{L1}^*$  - will be affected. The proportional change in the supply of  $X_1$  is now given by:

$$X_1^* = -[(\theta_{K3} S_1 \sigma^1 + S_2 \sigma^2 + S_3 \sigma^3) r^* + S_1 \sigma^1 t_{L1}^*] / |\lambda| \quad (12)$$

The tax will affect the supply of  $X_1$  only if  $\sigma^1 > 0$ ; otherwise, equations (10) and (12) will be identical.

Once again, equating  $X_1^*$  to the proportional change in the demand for  $X_1$  yields the solution for  $r^*$ :

$$r^* = [\epsilon \rho_{L1} |\lambda| + S_1 \sigma^1] t_{L1}^* / D \quad (13)$$

All the terms have been described above, including  $D$ , which is positive. The sign of  $r^*$ , therefore, depends on the numerator of equation (13). Because of the tax, the unit cost in  $X_1$  goes up, its output falls, and demand for its inputs is affected, as it did in the wake of the output tax considered earlier. The new element is the appearance of  $\sigma^1$  in the numerator of  $r^*$ , and it will have a noticeable effect on the incidence of this tax.

*Result 2: Labor can benefit from a partial tax on itself only if the taxed industry is relatively land intensive.*

For this outcome,  $r^*$  must be negative, and  $|\lambda| < 0$  provides a necessary condition. For large values of  $\sigma^1$ , however,  $r^*$  can still turn out to be positive. Labor normally benefits if the taxed industry is relatively land intensive because, other things being equal, as  $X_1$  contracts, a smaller excess supply of labor will ensue, but not if  $X_3$  can be substituted for labor with considerable ease. In the present case, from labor's standpoint, values of  $\sigma^1$  larger than  $(\epsilon p_{L1}|\lambda|/S_1)$  will outweigh any beneficial effect of a favorable configuration of land-labor ratios. For an output tax, a high elasticity of substitution in this industry had a more limited role. Since it appeared only in the denominator of  $r^*$  (in equation (11)), it could at most prevent a change in the wage-rental ratio, never causing it to reverse course.

*Result 3: If the taxed industry is relatively labor intensive, the mobile factor will suffer from a partial tax on labor.*

In this cases,  $|\lambda| > 0$ , and since  $\sigma^1 \geq 0$ ,  $r^*$  will be positive. The tax will lead to an excess supply of labor and the wage-rental ratio must fall to restore full employment. In this regard, this tax works very much like the partial tax on the output of this industry, and  $\sigma^1$  actually reinforces the effect of relative factor intensities.

*Result 4: If the taxed industry uses labor and the intermediate input in a fixed ratio, the wage-rental ratio will move in the same direction whether the tax is on output or labor.*

Comparing different taxes is often of interest in designing tax policy and for determining equivalent taxes. Results 1 and 2 together suggest that the wage-rental ratio can move in opposite directions for the two taxes if the taxed industry is relatively land-intensive. Fixed input-output coefficients essentially rule out this possibility. When  $\sigma^1 = 0$ , equations (11) and (13) will have

the same sign, determined by  $|\lambda|$ .

The conclusions about factor taxes, like the ones for output taxes, resemble some of the results in mobile-factor models where the incidence of partial factor taxes does depend on factor intensities and  $\sigma$ 's, as in the present setup by and large. The general outcome for the incidence of this tax in the 3x2 fgo model is that the mobile factor loses while the specific input in the untaxed sector gains, which may happen in our model also in some cases, but there is also the possibility of a benefit for the mobile factor (Result 2). Other aspects of these results, insofar as they rely on comparing input ratios, have no counterpart in the fgo model because, it was noted above, its specific inputs are generally unique to each sector and have no production linkages with the rest of the economy.

One result from that model, however, suggests an interesting elaboration of Results 2 and 4. McLure (1971) shows that the relative magnitudes of the elasticities of substitution and demand in the taxed industry decide its specific input's lot. In our notation,  $r^* \gtrless 0$  as  $\sigma \gtrless \varepsilon$ .<sup>3</sup> This is the second part of the well-known tax-incidence proposition mentioned in the Introduction. Its strict counterpart in our model is to ask how  $t_{L1}$  affects  $p_3$ , but that does not determine how the final burden of this tax is shared between the two primary factors of production. Now, if  $\varepsilon = 0$  in equation (13),  $r^*$  will be positive, and also when the tax is switched to  $X_2$  (in that case,  $r^* = (-\varepsilon p_{L2}|\lambda| + \lambda_{L2}\lambda_{K2}\sigma^2)t_{L2}^*/D$ ). It follows that, whether the mobile factor is taxed in one sector or the other, the primary specific input can gain or lose in both models albeit for very different reasons. In general, not much depends on the relative size of  $\sigma$  and  $\varepsilon$  here, and a completely inelastic demand will be sufficient to clinch the issue in favor of the specific input in either model. By contrast, a zero value for the elasticity of substitution in the taxed industry will make  $r^*$  negative in the McLure specification, whereas the wage-rental ratio can still increase or decrease in the psi

model. A selective output tax and a partial factor tax, however, will generate similar outcomes in the two models (Result 4).

What this elaboration really points to is the difficulty of defining equivalent specifications, or equivalent taxes for that matter, for it can be argued that  $t_{L2}$  should extend to  $X_3$  also in the psi model to be comparable with the corresponding tax in the 3x2 fgo case. The present treatment nonetheless can be defended by the argument that it is a tax on the mobile input directly employed in producing one of the two final goods in both models. Some aspects of model equivalence will be taken up in the next section where further comparisons with mobile-factor models are made. The numerical illustrations in Section 5 will also shed more light on this important issue.

#### 4. Comparisons with Mobile Factor Models

The closest model of this type is the two-sector Harberger model with both  $L$  and  $K$  mobile which, in Marshall's classification, will correspond to the long run, and some similarities in results have been noted already. For more precise comparisons, two questions raised in the Introduction are of particular interest: Can goods mobility totally offset the effects of factor immobility? If so, is the Marshallian short run still a useful concept? The main challenge is to find alternative specifications which may be regarded as comparable and equivalent. Empirical considerations will also be important because  $\lambda$ 's,  $\rho$ 's and  $\theta$ 's, which figure prominently in the expressions for  $r^*$ , are largely determined by the numbers in a given situation, and functional forms and elasticities will also matter. Analytical issues are taken up first, while questions of a more empirical nature will be considered along with the numerical illustrations in the next section.

#### 4.1 Goods mobility and factor immobility

One way of specifying a comparable mobile-factor model is to let  $X_1$  directly employ  $L_3$  and  $K_3$  in lieu of  $X_3$ , so that  $X_1 = f(L_1, L_3, K_3)$ , and then ensure that land rents are equalized between the two sectors. The corresponding goods-mobile production function can be written as  $X_1 = f(L_1, g(K_3, L_3))$ , where  $g(\bullet)$  is the production technology for  $X_3$ . The two specifications will be equivalent if  $\sigma_{L_1 L_3}$  is restricted to be equal to  $\sigma_{L_1 K_3}$ . Consequently, if the results are not very sensitive to these elasticity restriction, goods mobility can be regarded as an adequate antidote for the effects of immobile factors of production.

Another possibility, somewhat easier to handle analytically, is to add  $L_1$  and  $L_3$  (homogeneous labor), so that  $X_1$  becomes a simple two-input production function like the one used in the paper thus far, except that  $K_3$  replaces  $X_3$  and  $L_1$  also includes labor indirectly used for producing  $X_1$ . This requires a modification of the full employment and zero-profit conditions set out in Section 2, along with the definition of  $\sigma^1$  ( $r^*$  replaces  $p_3^*$ ).

For analytical purposes, these modifications essentially transform the psi model into the 2x2 mfo specification in Harberger (1962). The corresponding expressions for  $r^*$  (Mieszkowski (1967)) are very similar to equations (11) and (13), so the results formally derived there will apply here as well. Two qualifications nonetheless are worth noting: First, when all inputs are mobile and used directly in producing the two final goods,  $|\lambda|$  may have a different sign than in the produced-input specification, a type of factor-intensity reversal. In other words, when  $X_3$  is used by  $X_1$ , its land-labor ratio is smaller than in  $X_2$ , but factor intensities get reversed when all inputs are mobile and used directly for producing  $X_1$  and  $X_2$ . Second, when  $t_{L1}$  is considered,  $\sigma^1$  is the elasticity of substitution between labor and land in the mfo case, rather than between labor and  $X_3$ . Although both are defined to be non-negative,  $\sigma_{LK}^1$  and  $\sigma_{L3}^1$  need not be equal. They may be pre-multiplied



by different terms anyhow in the solutions for  $r^*$ . Therefore, the two specifications may produce different results from a given data set (one such outcome will be highlighted in Section 5.1 below). These are empirical matters for the most part, and it is doubtful that further manipulation of the underlying equations will lead to any useful generalizations.

## 4.2 Short run versus the long run

The Marshallian long run, strictly defined as the absence of immobile inputs, will occur in the present model when farm land actually becomes mobile. That will lead to a 2x3 framework - two mobile primary inputs and three goods, only two of which are needed for final demand - or it can be characterized as a 2x2 mobile factor setup with a pure intermediate product that is used for producing one of the final goods.

Under either characterization, the production function for  $X_1$  becomes:  $X_1 = f(K_1, L_1, X_3)$ , and  $R_{K1} = a_{K1} + a_{K3} a_{31}$ , to reflect total usage of land, analogous to  $R_{L1}$  for labor. The full employment condition for land can be written as  $R_{K1}X_1 + a_{K2}X_2 = \bar{K}$ , and  $\theta_{K1}$  in the zero-profit condition (equation (8)) is computed from  $R_{K1}$ . Labor and land thus are treated alike. The expanded production function will introduce some new (partial) elasticities of substitution,  $\sigma_{LK}^1$  and  $\sigma_{K3}^1$ , to reflect new margins of substitution, and there is the possibility of complementary inputs ( $\sigma < 0$ ) which does not arise in a two-input production function.<sup>4</sup>

From a technical standpoint, this three-input function is not comparable with the two-input production functions deployed thus far. Therefore, instead of attempting precise comparisons with the results noted above, we shall briefly explore the long-run specification on its own terms. The model can still be solved for  $r^*$  as before, and we focus on  $t_{L1}$  because a partial output tax will simply reiterate the role of relative factor intensities.

With  $t_{L1}$  in place, cost-minimizing firms will try to substitute  $K_1$  and  $X_3$  for labor if they can. Although some steps in the derivation become more complex because of the extra elasticities of substitution, the solution process remains intact, and

$$r^* = [A\epsilon\rho_{L1} + \gamma\rho_{L1}(\rho_{K1}\sigma_{LK}^1 + \rho_{31}\theta_{K3}\sigma_{L3}^1)]t_{L1}^* / D_1 \quad (14)$$

where  $A = (L_1/L_2 - K_1/K_2)$ , incorporates the direct and indirect usage of all inputs,  $\gamma = (\theta_{L1}\lambda_{K1}/\lambda_{K2} + \theta_{K1}\lambda_{L1}/\lambda_{L2})/\theta_{L1}\theta_{K1}$ , and the denominator,  $D_1$ , is very similar to  $D$  in the earlier equations. Both  $\gamma$  and  $D_1$  will be positive; therefore the sign of  $r^*$  once again will be determined by the terms in the numerator. Compared to equation (13), there is an additional term in the numerator of  $r^*$  which reflects possibilities of substituting labor for  $X_3$ . We can now write down some long-run results.

*Result 5: So long as the taxed industry is relatively labor intensive, and there are no complementary inputs, land owners will benefit from a partial tax on labor in  $X_1$ .*

In this situation,  $A$ ,  $\sigma_{LK}^1$  and  $\sigma_{L3}^1$  are all positive, so  $r^* > 0$ . The tax definitely leads to an excess supply of labor and hence a lower wage-rental ratio, unless one of the  $\sigma$ 's turns out to be negative - say, when labor and the intermediate good are complements - in which case  $r^*$  may be zero or negative.

*Result 6: The larger is the elasticity of substitution between land and the intermediate good in the taxed industry, the greater will be the tendency for the wage-rental ratio to remain unchanged.*

Land, now mobile between the two industries, is the untaxed primary factor of production. The relevant elasticity,  $\sigma_{K3}^1$ , is an element of  $D_1$ ; it does not appear in the numerator of (14), therefore it cannot affect the sign of  $r^*$ . In the limit, as  $\sigma_{K3}^1 \rightarrow \infty$ ,  $r^* \rightarrow 0$ , which implies that labor and land will bear the burden of this tax in proportion to their initial factor shares.

Other results of this sort can be written down, by selecting extreme values of the elasticities, for instance (fixed proportions, "large" values of  $\sigma_{L3}^1$ , and so on), but most of them will have strong empirical overtones. As in Result 2, the factor-intensity term and the ones involving the elasticities of substitution must work at cross purposes to generate any possibilities for switching the sign of  $r^*$ . That implies either some complementary inputs or a taxed industry which uses land relatively intensively. Otherwise, only the magnitude of  $r^*$  will change. Instead of further exploring these possibilities analytically, we shall consider some plausible numerical examples in the next section.

All in all, the tax incidence results in the 3x2 model developed here seem to share many aspects of the results in mobile-input models, but only rarely will they coincide, and under rather severe restrictions on the underlying parameters. By the same token, the Marshallian short-run-long-run distinction will continue to matter even though some of the analytical propositions have a touch of the long run. The results, especially for output taxes, would not alter much if factor-intensity ranks of the final-good industries remain unchanged under different specifications. It remains to be seen whether analytically equivalent specifications will generate the exact same empirical outcomes.

## 5. Numerical Illustrations

These computations are based on the stylized U. S. data in Solow (1987) which are rearranged to depict the three models discussed above in a consistent manner. The goal is to illustrate some of the analytical results for a range of plausible values for the various parameters, especially those where the theoretical discussion points to an uncertain outcome.

CES production and utility functions are specified in each case for calibrating the models

to the benchmark data, and the algorithm in Rutherford (1988) is used to compute a new equilibrium for each tax under the assumptions of fixed factor endowments and perfect competition all round. We shall consider a "small" tax, one percent of the relevant tax base in each case, so  $r^*$ 's can be viewed as tax-elasticities of relative factor prices. The assumptions, the functional forms, and the solution algorithm are consistent with the models considered here, so these illustrations should reflect the essential features of the analytical solutions presented in the paper.<sup>5</sup> The numbers indicate that  $X_1$  is relatively labor intensive, which implies  $|\lambda| > 0$ .

<Table 1>

The basic data for the psi model are presented in Table 1, and the first task is to set up comparable numerical renditions of the other two models. The 3x2 specification in McLure (1971) can be approximated by assuming that  $X_2$  and  $X_3$  require different types of land, and the latex land is used directly for producing  $X_1$  in one integrated operation. The  $X_1$  column in Table 1 will then show 198.0 for  $K$  instead of the row entry for  $X_3$ , and labor employed in  $X_2$  will increase to 201.98. The land rents,  $r_1$  and  $r_2$ , will be determined as a residual in each industry. These numbers will also work for the 2x2 mobile factor case, except that land will earn the same rent everywhere. In both cases  $\sigma^1$  will be the elasticity of substitution between labor and land, rather than between labor and  $X_3$  as in the psi setup. In each of the three specifications, thus, the two final-good industries use the same amount of land, directly or indirectly, in the initial equilibrium, whereas the labor input would differ.<sup>6</sup> Therefore, while considering a tax on labor, the tax rates will be adjusted where necessary to ensure equal tax revenues.

Table 2 illustrates the results for taxes in  $X_1$ , and since this industry is relatively labor intensive, the wage-rental ratio falls in every case, as the analytical results in the paper indicate. In the fgo computations, the signs of  $r_1^*$  and  $r_2^*$  are also what the McLure model predicts (the

immobile input in the untaxed industry always benefits). One can therefore have confidence in the data and the computational procedures while approaching more complex analytical propositions such as Result 2 where the outcome is uncertain when the taxed industry is relatively land intensive. In this data set,  $X_2$  is that industry, and  $t_{L2}$  is the relevant tax. Recall that in this case,  $r^* = (-\epsilon_{p_{L2}}|\lambda| + \lambda_{L2}\lambda_{K2}\sigma^2)t_{L2}^*/D$ , which will be negative except for "large" values of  $\sigma^2$ .

<Table 2 here>

Both negative and positive  $r^*$ 's appear in Table 3, and they get smaller and smaller as the elasticity of substitution in the taxed industry drops. Values of  $\sigma^2 \geq 0.5$  seem to be large enough to make  $r^*$  positive, though, which is surprising because such estimates of  $\sigma$  are often cited as examples of low substitution elasticities in empirical work. The mfo specification replicates this pattern, although in four of the six cases,  $r^*$ 's are twice the size of the ones in the psi model. Low substitution elasticities in the taxed sector hurt its primary specific factor in the 3x2 fgo specification as well:  $r_2^* < 0$  in two of the three cases ( $\sigma^2 \leq 0.5$ ). The  $r^*$ 's turn out to be identical in the three models when all elasticities are unity, but in other situations, the fgo specification is the worst from the land-owner's point of view.

<Table 3 here>

### 5.1 Goods mobility and elasticity restrictions

Regarding "equivalent" models, Tables 2 and 3 already provide some examples because the mfo production function, with  $K$  and  $L$  as inputs, and the psi production function relying on  $L$  and  $X_3$ , meet the equivalence criterion set out in Section 4.1. For the three taxes considered in these tables, the  $r^*$ 's always agree in sign although rarely in magnitude. Even when  $\sigma_{L3}^1 = \sigma_{LK}^1$ , the computed tax elasticities are not the same (e.g., rows (b) and (c), top two panels in Table 3). This

was one of the possible outcomes noted at the end of Section 4.1 above.

For an alternate comparison, the data are rearranged such that the land-labor ratios in the pre-tax equilibrium are the same in the two specifications, i.e. there is a common starting value for  $|\lambda|$  rather than for  $K$ , the assumption underlying Tables 2 and 3. Accordingly, the  $K, L$  numbers in the  $X_3$  column are transferred to the  $X_1$  column in Table 1, and the row entry for  $X_3$  is deleted. The land-labor ratios for  $X_1$  and  $X_2$  in the mfo specification now are further apart (a larger and still positive value for  $|\lambda|$ ) than in the earlier tables, and its effect is highlighted in case (c) ( $\sigma^2 = 0.3$ ) where, going from Table 3 to Table 4,  $r^*$  drops from -0.0002 to -0.0006. Labor benefits a good deal from this tax in this situation. Similar changes occur in other instances too, but the new assumption does not alter the sign of  $r^*$  even once.

Some computations incorporating a three-input production function, with and without the elasticity restrictions discussed in Section 4.1 are also reported in Table 4 (lower panel): in one case, for  $t_{L1}$   $r^*$  drops from .0083 to .0025 but increases from 0.0018 to 0.0025 for  $t_{L2}$ . The elasticity restrictions, therefore, do seem to matter. Considering all the computations together, the overall conclusion must be that different versions of the production function for  $X_1$  do not produce the same numerical results with this data set, except for a partial tax on labor when all  $\sigma$ 's are equal to one.

<Table 4 here>

## 5.2 Short-run-long-run considerations

The long-run production function (Section 4.2) is put to work for the computations in Table 5, assuming that one half of  $K_3$  and  $L_3$  are directly employed in  $X_1$ .<sup>7</sup>

<Table 5 here>

The wage-rental ratio appears to be positively correlated with the elasticity of substitution in the taxed industry for both factor taxes. As Result 5 in the paper suggests,  $r^*$  is positive in case of  $t_{L1}$ . For  $t_{L2}$ , limited possibilities of input substitution in  $X_2$  are good for labor because, other things equal, the tax leads to a smaller excess supply of labor, and in row (c), the wage-rental ratio actually increases. Effects of assuming fixed input-output coefficients (row (d)) are of some interest: if  $X_3$  cannot be substituted for land or labor in  $X_1$ , the wage-rental ratio increases by about 14 percent when labor directly employed in that industry is taxed, whereas it drops by about 19 percent if the tax is switched to  $X_2$ . Relative to the tax elasticities reported earlier, a logical comparison is with the numbers in the lower panel of Table 4, because both production functions have three inputs and the same total usage of land and labor, directly or indirectly, and the  $r^*$ 's are different in every case, even when all elasticities are unity.

In summary, the range of tax elasticities reported in the tables in this section illustrate many of the analytical propositions derived in the paper. In some instances, they help in quantifying the qualitative aspects of the results (e.g. Result 2), and they also highlight the complications which arise in setting up the empirical counterparts to analytically equivalent specifications. A common theme running through many of the computations is that equivalent models do not always generate the same numerical outcomes. The  $r^*$  values can diverge considerably, especially when elasticities other than unity are involved. Complementary inputs and possibilities of factor-intensity reversal, not considered here mainly to limit the length of this section, would change the results even more.

## 6. Conclusions

This paper has developed a framework for analyzing the role of sector-specific inputs which have a definite value-added component, a large and expanding class of inputs ranging from simple,

primary commodities to sophisticated high-tech products. Starting with the classic 3x2 fgo setup (McLure (1971)), the value-adding process as well as the cross-sector connections of immobile inputs are explicitly modeled. A small, neo-classical general equilibrium model is deployed to examine the incidence of various taxes, and the analytical propositions are illustrated, and refined in some cases, with the help of cge examples from stylized U.S. data. Goods mobility does seem to offset the effects of factor immobility, and several results reflect many aspects of tax models with only mobile factors of production, but tax elasticities computed from the data are rarely the same. By the same token, the Marshallian short-run-long-run distinction, blurred in some ways by the production linkages modeled here, does not disappear.

Although the framework and the new results provide ample justification for revisiting a tax model from the 1970's, in our view, some may find this model dated and the issues somewhat archaic. A straight numerical approach, based on cge or other techniques, may be quicker to reach the final answers, but the blend of analytical and empirical methods attempted here is better than either approach in isolation. The tax literature does not have many examples of this sort in which theoretical results from inter-related models are compared, contrasted and then numerically illustrated in a consistent manner. There is nothing outmoded about the issues of goods mobility and factor immobility, short-run versus long-run, and model equivalence generally, especially the intertwining of its theoretical and empirical aspects highlighted in this paper.



## Endnotes

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<sup>1</sup>The elasticity of substitution in  $X_i$  is defined as  $\sigma^i = (a_{Ki}^* - a_{Li}^*)/(w^* - r^*)$ . The expressions for  $R^*$  and  $a_{ij}^*$  follow from these definitions and the fact that in the type of production functions specified here, at minimum-unit cost, the share-weighted average of  $a_{ij}^*$  in each industry is zero. See Jones (1971) for more details.

<sup>2</sup>This demand function has been widely used in the tax literature. It is based on the assumptions that the tax revenue is always returned in a lump-sum fashion, and the income elasticities of demand for the two final goods are unity.

<sup>3</sup>More precisely,  $(r_1^* - w^*) = [\rho_{L1}\sigma^2(\sigma^1 - \varepsilon) + \varepsilon\sigma^1\rho_{K2}(K_1/K_2)]t_{L1}^*/D$ , in our notation, after switching the numeraire from  $p_2$  to  $w$  in McLure (1971), Table I, Case 3. Thus,  $\sigma^1 \geq \varepsilon$  or  $\varepsilon = 0$  will be sufficient to make  $(r_1^* - w^*)$  positive. By contrast,  $(r_1^* - w^*) < 0$  if  $\sigma^1 = 0$ .

<sup>4</sup>The partial elasticity of substitution between, say, labor and land in  $X_1$  is defined as:

$$\rho_{L1}\sigma_{LK}^1 = \frac{\partial a_{K1}}{\partial w} \frac{w}{a_{K1}}.$$

Complementarity implies that  $\partial a_{K1}/\partial w = 0$ . As Allen (1967) shows, in a three-input production function of the type specified here, at most only one of the  $\sigma$ 's can be negative.

<sup>5</sup>An alternative approach, followed by Harberger (1962) and others, would be to calculate factor shares and land-labor ratios from the data in Table 1 and then plug them in to the expressions for  $r^*$  such as equation (13). The cge approach is preferable in this regard because it allows factor shares to adjust in response to the taxes; in fact, a new equilibrium, complete with a set of new prices, is computed at each step.

<sup>6</sup>This seems appropriate, given the focus of this analysis on specific inputs, but other specifications, such as having the same number of workers or the same land/labor ratios, may be

equally valid. See Table 4 for some computations based on equal land-labor ratios.

<sup>7</sup>This reallocation is consistent with the assumption just discussed about land-labor ratios in Table 4, although it is not based on any structural features or other details of the economy depicted by these data.

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**Table 1**  
**The Benchmark Data<sup>a</sup>**

	$X_1$	$X_2$	$X_3$	$C$
$X_1$				870.75
$X_2$				437.89
$X_3$	270.33			
$K$	--	308.24	198.00	
$L$	600.42	129.65	72.33	
	870.75	437.89	270.33	1,308.64

<sup>a</sup>The  $X$ 's are output levels.  $C$  is the consumption of each final good, and  $K$  and  $L$ , respectively, denote capital and labor employed in each industry.

**Table 2**

**Effect of Taxes in  $X_1$  on Wage-Rental Ratios<sup>a</sup>**

PSI					
		$T_1$		$t_{L1}$	
		$r^*$			
(a)	All $\sigma$ 's = 1.0	.0045		.0075	
(b)	$\sigma_{L3}^1 = 0.5$ , other $\sigma$ 's = 1.0	.0053		.0063	
(c)	$\sigma_{L3}^1 = 0.3$ , other $\sigma$ 's = 1.0	.0058		.0057	
2x2 MFO					
		$T_1$		$t_{L1}$	
		$r^*$			
(a)	All $\sigma$ 's = 1.0	.0036		.0075	
(b)	$\sigma_{LK}^1 = 0.5$ , other $\sigma$ 's = 1.0	.0047		.0062	
(c)	$\sigma_{LK}^1 = 0.3$ , other $\sigma$ 's = 1.0	.0054		.0067	
3x2 FGO					
		$r_1^*$	$r_2^*$	$r_1^*$	$r_2^*$
(a)	All $\sigma$ 's = 1.0	-.0025	.0075	.0075	.0075
(b)	$\sigma^1 = 0.5$ , other $\sigma$ 's = 1.0	-.0047	.0070	.0053	.0071
(c)	$\sigma^1 = 0.3$ , other $\sigma$ 's = 1.0	-.0073	.0065	.0065	.0066

<sup>a</sup>The numbers correspond to  $r^*$  in the analytical results derived in the paper. Each tax is levied at the rate of one percent in  $X_1 - T_1$  on the value of output, and  $t_{L1}$  on the earnings of labor directly employed in the industry. The model developed in the paper is PSI, 2x2 MFO refers to the Harberger mobile-factor model (1962), and the 3x2 FGO specification is from McLure (1971). The  $\sigma$ 's denote elasticities of substitution.

**Table 3**

**Effect of a Wage Tax in  $X_2(t_{L2})$  on Wage-Rental Ratios<sup>a</sup>**

PSI		$r^*$	
(a)	All $\sigma$ 's = 1.0	.0016	
(b)	$\sigma_{L3}^1 = 0.5$ , other $\sigma$ 's = 1.0	.0002	
(c)	$\sigma_{L3}^1 = 0.3$ , other $\sigma$ 's = 1.0	-.0001	
2x2 MFO <sup>b</sup>		$r^*$	
(a)	All $\sigma$ 's = 1.0	.0016	
(b)	$\sigma_{LK}^1 = 0.5$ , other $\sigma$ 's = 1.0	.0004	
(c)	$\sigma_{LK}^1 = 0.3$ , other $\sigma$ 's = 1.0	-.0002	
3x2 FGO			
		$r_1^*$	$r_2^*$
(a)	All $\sigma$ 's = 1.0	.0016	.0016
(b)	$\sigma^1 = 0.5$ , other $\sigma$ 's = 1.0	.0011	-.0002
(c)	$\sigma^1 = 0.3$ , other $\sigma$ 's = 1.0	.0008	-.0014

<sup>a</sup>The  $r^*$  values correspond to the solutions derived in the paper. The tax rate is one percent of the wage earnings in  $X_2$ . The  $\sigma$ 's denote elasticities of substitution.  $X_1$  and  $X_2$  use the same quantity of land in the pre-tax equilibrium in each case.

<sup>b</sup>In the initial equilibrium,  $X_2$  employs 201.98 units of labor, but only 129.65 in the PSI specification. The tax rate has been adjusted to yield the same tax revenue in the two cases.

**Table 4**

**Effect of a Wage Tax on the Wage-Rental Ratio<sup>a</sup>**

Production function <sup>b</sup> : $X_1 = f(L_1, K_3)$		
	$t_{L1}$	$t_{L2}$
(a) All $\sigma$ 's = 1.0	.0079	.0016
(b) Taxed industry $\sigma = 0.5$ , other $\sigma$ 's = 1.0	.0074	.0002
(c) Taxed industry $\sigma = 0.3$ , other $\sigma$ 's = 1.0	.0071	-.0006
Production function: $X_1 = f(L_1, L_3, K_3)$		
	$t_{L1}$	$t_{L2}$
(a) All $\sigma$ 's = 1.0	.0083	.0018
(b) Taxed industry $\sigma$ 's = 0.5, other $\sigma$ 's = 1.0	.0076	.0003
(c) Taxed industry $\sigma$ 's = 0.3, other $\sigma$ 's = 1.0	.0071	-.0005
(d) $\sigma_{L1L3}^1 = \sigma_{L1K3}^1 = 0.5$ , other $\sigma$ 's = 1.0 <sup>c</sup>	.0025	.0025
(e) $\sigma_{L1L3}^1 = \sigma_{L1K3}^1 = 0.3$ , other $\sigma$ 's = 1.0	.0071	.0030

<sup>a</sup>The  $r^*$ -values correspond to the solutions derived in the paper.

<sup>b</sup>The initial land-labor ratio is the same as in the PSI specification in Table 3.

<sup>c</sup>See Section 4.1 for a discussion of restrictions on  $\sigma$ 's.

**Table 5**

**Effect of a Wage Tax on the Wage-Rental Ratio: The Long-Run Case<sup>a</sup>**

Production function: $X_1 = f(K_1, L_1, X_2)$		
	$t_{L1}$	$t_{L2}$
(a) All $\sigma$ 's = 1.0	$r^* = .0080$	.0016
(b) All $\sigma$ 's in the taxed industry = 0.5, other $\sigma$ 's = 1.0	.0071	.0002
(c) All $\sigma$ 's in the taxed industry = 0.3, other $\sigma$ 's = 1.0	.0066	-.0006
(d) $\sigma_{L3}^1 = \sigma_{K3}^2 = 0$ , $\sigma_{LK}^1 = \sigma_{LK}^2 = 1.0$	.0070	.0019

<sup>a</sup>The  $r^*$ -values correspond to the expressions derived in the paper (e.g., equation (14) for  $t_{L1}$ ). The tax rate is one percent of the earnings of labor directly employed in the taxed industry.